THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA
KAPREKAR CONTEST – FINAL – SUB JUNIOR

Classes VII & VIII

Saturday, 2nd October, 2016

Instructions:

1. Answer as many questions as possible.
2. Elegant and novel solutions will get extra credits.
3. Diagrams and explanations should be given wherever necessary.
4. Fill in FACE SLIP and your rough working should be in the answer book.
5. Maximum time allowed is THREE hours.
6. All questions carry equal marks.

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1. (a) If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = 2016$, where x, y, z, a, b, c are non zero real numbers, find the value of

$$\frac{xyz(a+b)(b+c)(c+a)}{abc(x+y)(y+z)(z+x)}$$

(b) Four boys Amar, Benny, Charan, Dany, four boys and four girls Azija, Beula, Chitra and Dais have to work on a project. They should from 4 pairs, one boy and one girl in each. They know each other with the following constraints:

i. Amar knows neither Azija nor Buela

ii. Benny does not know Buela

iii. Both Charan and Dany know neither Chitra nor Daisy.

In how many ways can the pairs be formed so that each boy knows the girl in his pair.

2. In a triangle ABC, $\angle C = 90^\circ$ and $BC = 3AC$. Points D, E lie on CB such that $CD = DE = EB$. Prove that

$$\angle ABC + \angle AEC + \angle ADC = 90^\circ$$

3. Let m, n, p be distinct two digit natural numbers. If

$$m = 10a + b, \quad n = 10b + c, \quad p = 10c + a$$

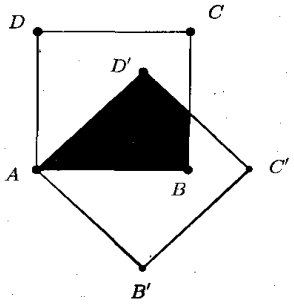
find all possible values of $GCD(m, n, p)$.

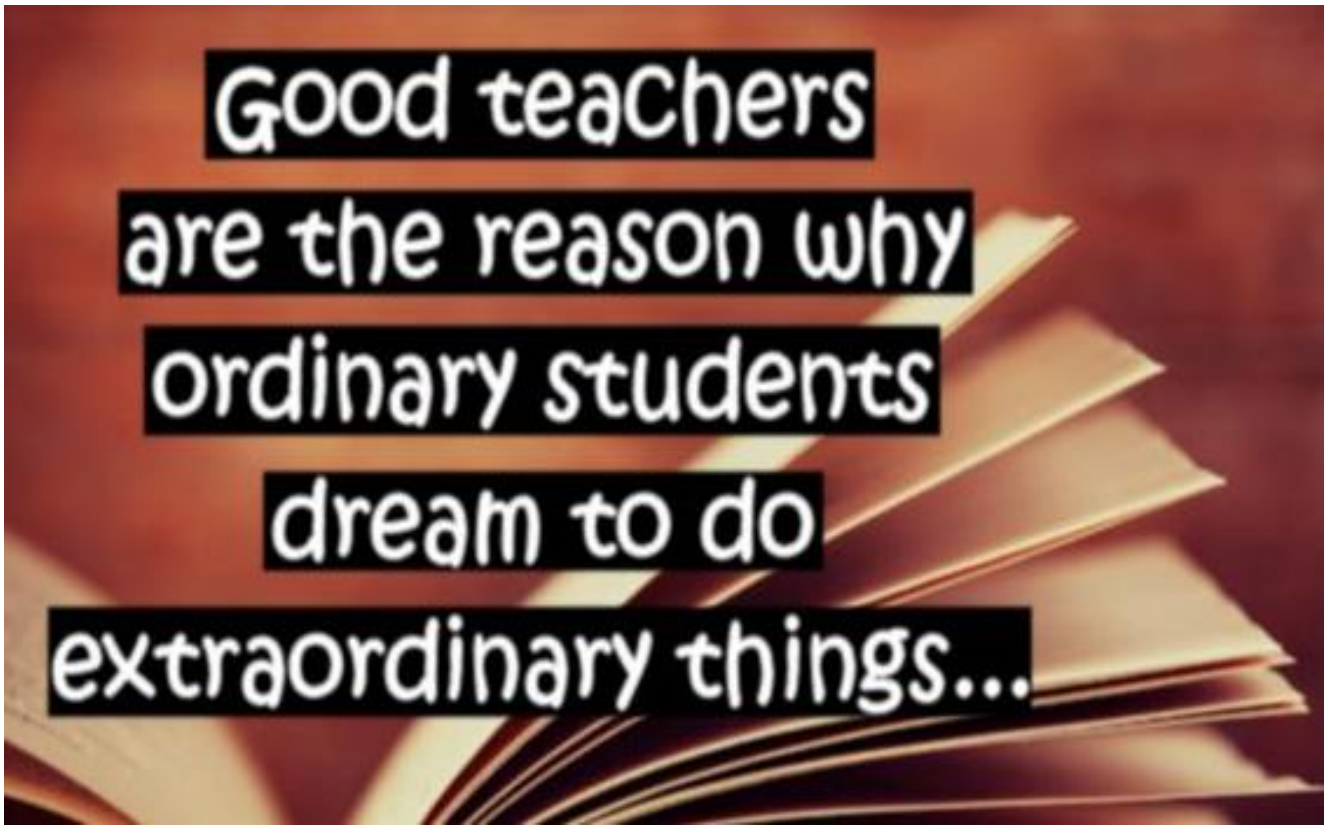
4. If $xy = ab(a+b)$ and

$$x^2 + y^2 - xy = a^3 + b^3$$

find the value of $\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{b} - \frac{y}{a}\right)$

5. The square $ABCD$ of side length a cm is rotated about A in the clockwise direction by an angle 45° to become the square $AB'C'D'$. Show that the shaded area is $(\sqrt{2} - 1)a^2$ square cms.





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