NMTC Final - SJ
Year 2017 Test Paper

**1.** **(a)** Find all three digit numbers in which any two adjacent digits differ by 3.

**(b)** There are 5 cards. Five positive integers (may be different or equal) are written on these cards, one on each card. Abhiram finds the sum of the numbers on every pair of cards. He obtains only three different totals 57, 70, 83. Find the largest integer written on a card.

**2.** **(a)** ABC is a triangle in which AB = 24, BC = 10 and CA = 26 . P is a point inside the triangle. Perpendiculars are drawn to BC, AB and AC. Length of these perpendiculars respectively are x, y and z. Find the numerical value of $5x + 12y + 13z$.

**(b)** If $x^2(y + z) = a^2$, $y^2(z + x) = b^2$, $z^2(x + y) = c^2$, $xyz = abc$ prove that $a^2 + b^2 + c^2 + 2abc = 1$

**3.** If

$$X = \frac{a^2 - (2b-3c)^2}{(3c+a)^2 - 4b^2} + \frac{4b^2 - (3c-a)^2}{(a+2b)^2 - 9c^2} + \frac{9c^2 - (a-2b)^2}{(2b+3c)^2 - a^2}$$

$$Y = \frac{9y^2 - (4z-2x)^2}{(2x+3y)^2 - 16z^2} + \frac{16z^2 - (2x-3y)^2}{(3y+4z)^2 - 4x^2} + \frac{4x^2 - (3y-4z)^2}{(4z+2x)^2 - 9y^2}$$

Find 2017 (X + Y)

**4.** The sum of the ages of a man and his wife is six times the sum of the ages of their children. Two years ago the sum of their ages was ten times the sum of the ages of their children. Six years hence the sum of their ages will be three times the sum of the ages of their children. How many children do they have?

**5.** **(a)** a, b, c are three natural numbers such that $a \times b \times c = 27846$. If $\frac{a}{6} = b + 4 = c - 4$, find a + b + c.

**(b)** ABCDEFGH is a regular octagon with side length equal to a. Find the area of the trapezium ABDG.

**6.** **(a)** If a, b, c are positive real number such that no two of them are equal, show that $a(a-b)(a-c) + b(b-c)(b-a) + c(c-a)(c-b)$ is always positive

**(b)** In the figure below, P, Q, R, S are point on the sides of the triangle ABC such that CP = PQ = QB = BA = AR = RS = SC